Voros product, noncommutative inspired Reissner-Nordström black hole and corrected area law

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Abstract

We emphasize the importance of the Voros product in defining a noncommutative inspired Reissner-Nordström black hole. The entropy of this black hole is then computed in the tunneling approach and is shown to obey the area law at the next to leading order in the noncommutative parameter θ. Modifications to entropy/area law is then obtained by going beyond the semi-classical approximation. The leading correction to the semiclassical entropy/area law is found to be logarithmic and its coefficient involves the noncommutative parameter θ.

Classical general relativity gives the concept of black hole from which nothing can escape. This picture was changed dramatically when Hawking [1, 2] incorporated the quantum nature into this classical problem. He showed by combining gravity and quantum mechanics that black holes emit a spectrum that is similar to a thermal black body spectrum. This result made the *first law of black hole mechanics* [3] closely analogous to the first law of thermodynamics. This analogy ultimately led to the entropy for black holes and was consistent with the proposal made

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by Bekenstein [4, 5, 6, 7] that a black hole has an entropy proportional to its horizon area. All the above issues finally led to the famous Bekenstein-Hawking area law for the entropy of black holes given by $S_{BH} = A/4$.

However, most of these calculations were based on a semiclassical treatment and also on a commutative spacetime. The standard Bekenstein-Hawking area law is known to get corrections due to quantum geometry or back reaction effects [8]. Recently, modifications to the semiclassical area law due to noncommutative (NC) spacetime have also been obtained [9]-[14]. The motivation for these investigations was that noncommutativity is expected to be relevant at the Planck scale where it is known that usual semiclassical considerations break down.

In a recent paper [13], it has been pointed out that the Voros star product [15],[16] plays a key role in the obtention of the mass density of a static, spherically symmetric, smeared, particle-like gravitational source required in getting the NC inspired Schwarzschild black hole [17],[18] [1]. Quantum corrections to the semiclassical Hawking temperature and entropy for NC inspired Schwarzschild black hole have been obtained next. This is done by first computing the correction to the Hawking temperature by going beyond the semiclassical approximation in the tunneling method [32]-[38]. Using the corrected form of the Hawking temperature and the first law of black hole thermodynamics, the entropy is computed. The result is seen to contain logarithmic and inverse horizon area corrections and holds to $\mathcal{O}(\sqrt{\theta}e^{-M^2/\theta})$.

In this paper, we carry out the above analysis in the case of NC inspired Reissner-Nordström (RN) black hole. To obtain NC effects on the usual area law, computation of the (NC) Hawking temperature is carried out. The first law of thermodynamics is then used to obtain the entropy. It is observed that to the next to leading order in the NC parameter θ, in the regime $r_h^2/(4\theta) >> 1$, the (NC) area law is just a NC deformation of the usual semiclassical area law as in case of the NC inspired Schwarzschild black hole. Quantum corrections to the area law is then found by following the procedure as mentioned above for the NC inspired Schwarzschild black hole. The coefficient of the logarithmic correction term is explicitly determined from the trace anomaly of the stress tensor [39],[40]. This coefficient is also found to have NC correction.

To begin the discussion, we follow the analysis in [13] based on the formulational and interpretational aspects of NC quantum mechanics to highlight the important part played by the

[1] Note that there are ways in which the Moyal star product [19] is used to incorporate the NC effect in gravity [20]-[24]. The twisted formulation of NC quantum field theory [25]-[31] is yet another way of incorporating the effects of noncommutativity in gravity.

Voros star product in writing down the mass and charge densities of a static, spherically symmetric, smeared, particle-like charged gravitational source required in getting the NC inspired RN black hole

$$
\begin{aligned}
\rho_\theta^{(M)}(r) &= \frac{M}{(4\pi\theta)^{3/2}} \exp\left(-\frac{r^2}{4\theta}\right) \\
\rho_\theta^{(Q)}(r) &= \frac{Q}{(4\pi\theta)^{3/2}} \exp\left(-\frac{r^2}{4\theta}\right) .
\end{aligned} \tag{1}
$$

Solving Einstein's equations with the above mass and charge densities incorporated in the energy-momentum tensor leads to the following NC inspired RN metric [10],[12]

$$
ds^2 = -f_\theta(r)dt^2 + f_\theta^{-1}(r)dr^2 + r^2(d\tilde{\theta}^2 + \sin^2\tilde{\theta}d\phi^2) \tag{2}
$$

where

$$
\begin{aligned}
g_{tt}(r) &= g^{rr}(r) = f_\theta(r) = 1 - \frac{4M}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) + \frac{Q^2}{\pi r^2}\left[F(r) + \sqrt{\frac{2}{\theta}}r\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)\right] \\
F(r) &= \gamma^2\left(\frac{1}{2}, \frac{r^2}{4\theta}\right) - \frac{r}{\sqrt{2\theta}}\gamma\left(\frac{1}{2}, \frac{r^2}{2\theta}\right).
\end{aligned} \tag{3}
$$

The event horizon of the black hole can be found by setting $g_{tt}(r_h) = 0$ in (2). Since this equation cannot be solved in a closed form, we take the large radius regime ($\frac{r_h^2}{4\theta} >> 1$) where we can expand the incomplete gamma function to solve r_h by iteration. Keeping upto the next to leading order $\frac{1}{\sqrt{\theta}}e^{-\frac{r_0^2}{4\theta}}$, we find

$$
r_h \simeq r_0 \left[1 - \frac{r_0}{2\sqrt{\pi\theta}(r_0 - M)}\left(2M - \frac{Q^2}{\sqrt{2\pi\theta}}\right)e^{-r_0^2/(4\theta)}\right] \tag{4}
$$

where

$$
r_0 = M + \sqrt{M^2 - Q^2} \tag{5}
$$

is the horizon radius of the commutative RN black hole.

Now for a general static and spherically symmetric spacetime, the Hawking temperature (T_H) is related to the surface gravity (κ) by the following relation [41]

$$
T_H = \frac{\hbar\kappa}{2\pi} \tag{6}
$$

where the surface gravity of the black hole is given by

$$\kappa = \frac{1}{2}\left(\frac{df_\theta}{dr}\right)_{r=r_h} . \tag{7}$$

Therefore in the regime ($\frac{r_h^2}{4\theta} >> 1$), the Hawking temperature for the NC inspired RN black hole is found to be (upto order $\frac{1}{\sqrt{\theta}}e^{-r_0^2/(4\theta)}$)

$$\begin{aligned} T_H \;\; \simeq \;\; & \frac{\hbar}{2\pi r_0^3}\Bigg[Mr_0 - Q^2 + \frac{Mr_0^2}{\sqrt{\pi\theta}(r_0 - M)}\left(3M - r_0 - \frac{Q^2}{\sqrt{2\pi\theta}} - \frac{(r_0 - M)}{2\theta}r_0^2\right)e^{-r_0^2/(4\theta)} \\ & + \frac{Q^2 r_0^4}{\sqrt{2}4\pi\theta^2}e^{-r_0^2/(4\theta)}\Bigg] . \end{aligned} \tag{8}$$

We shall now use the first law of black hole thermodynamics to calculate the Bekenstein-Hawking entropy. The first law of black hole thermodynamics is given by [42]

$$S = \int \frac{dM}{T_H} + \int Y dQ \tag{9}$$

where

$$\begin{aligned} Y \;\; &= \;\; -\frac{\Phi_H}{T_H} - \frac{\partial}{\partial Q}\int \frac{dM}{T_H} \\ \Phi_H \;\; &= \;\; \frac{Q}{r_h} . \end{aligned} \tag{10}$$

Using eq.(8), it can be easily shown that $Y = 0$ for the NC inspired RN black hole. Hence the Bekenstein-Hawking entropy upto next to leading order in θ is found to be

$$S = \int \frac{dM}{T_H} = \pi r_0^2\left[1 - \frac{r_0}{\sqrt{\pi\theta}(r_0 - M)}\left(2M - \frac{Q^2}{\sqrt{2\pi\theta}}\right)e^{-r_0^2/(4\theta)}\right] . \tag{11}$$

In order to express the entropy in terms of the NC horizon area (A_θ), we use eq.(4) to obtain

$$A_\theta = 4\pi r_h^2 = 4\pi r_0^2\left[1 - \frac{r_0}{\sqrt{\pi\theta}(r_0 - M)}\left(2M - \frac{Q^2}{\sqrt{2\pi\theta}}\right)e^{-r_0^2/(4\theta)}\right] . \tag{12}$$

Comparing eq(s)(11, 12), we find that at the leading order in θ, the NC black hole entropy

satisfies the area law

$$S = S_{\rm BH} = \frac{A_\theta}{4\hbar} \ . \tag{13}$$

This is functionally identical to the Bekenstein-Hawking area law in the commutative space. Hence we have analytically observed that in the regime $\frac{r_h^2}{4\theta} >> 1$, the NC version of the semiclassical Bekenstein-Hawking area law holds upto leading order in θ. This motivates us to investigate the corrections to the semiclassical area law upto leading order in θ.

To do so, we first compute the corrected Hawking temperature $\tilde{T}_H$. Following the tunelling method in [32] which goes beyond the semiclassical approximation, one obtains the corrected Hawking temperature as

$$\tilde{T}_H = T_H \left[1 + \sum_i \frac{\tilde{\beta}_i \hbar^i}{(Mr_h - Q^2/2)^i}\right]^{-1} \ . \tag{14}$$

Now applying the first law of black hole thermodynamics once again with this corrected Hawking temperature, we obtain the following expression for the corrected entropy/area law :

$$\begin{aligned} S_{bh} &= \frac{A_\theta}{4\hbar} + 2\pi\tilde{\beta}_1 \ln A_\theta + \mathcal{O}(\sqrt{\theta} e^{-r_0^2/(4\theta)}) \\ &= S_{BH} + 2\pi\tilde{\beta}_1 \ln S_{BH} + \mathcal{O}(\sqrt{\theta} e^{-r_0^2/(4\theta)}) \end{aligned} \tag{15}$$

where A_θ and S_{BH} are defined in (12) and (13) respectively. This expression is functionally identical to the corrected entropy/area law for the standard Schwarzschild black hole [35, 42]. However there is an important difference. This expression of corrected entropy has both noncommutative and quantum corrections. Although here we have restricted ourselves only to the leading order correction due to the NC parameter (θ), one can try to include all order θ corrections. This is technically more involved and we shall not address this issue in this paper.

Now we move on to compute the coefficient $\tilde{\beta}_1$ in the above expression. By making an infinitesimal scale transformation to the metric coefficients in (2), the coefficient $\tilde{\beta}_1$ can be

related to the trace anomaly in the following way [42]:

$$\begin{aligned}
\tilde{\beta}_1 &= -\frac{\sqrt{M^2-Q^2}}{4\pi\omega}\mathrm{Im}\int d^4x\ \sqrt{-g}\ \langle T^\mu{}_\mu\rangle^{(1)} \\
&= -\frac{(r_0-M)}{4\pi\omega}\mathrm{Im}\int_{r_h}^{\infty}\int_0^{-i\beta}\int_0^{\pi}\int_0^{2\pi} r^2\sin\tilde{\theta}\langle T^\mu{}_\mu\rangle^{(1)} dr dt d\tilde{\theta} d\phi \\
&= \frac{(r_0-M)}{2880\pi^2\omega}\beta\int_{r_h}^{\infty} r^2\langle T^\mu{}_\mu\rangle^{(1)} dr\ . \qquad (16)
\end{aligned}$$

Here, $\langle T^\mu{}_\mu\rangle^{(1)}$ is the trace anomaly calculated for the first loop expansion and ω is given by the Komar energy integral

$$\omega = \frac{1}{4\pi}\int_{\partial\Sigma} d^2x\ \sqrt{p^{(2)}}\ n^\mu\sigma^\nu\nabla_\mu K_\nu \qquad (17)$$

evaluated near the event horizon. The one loop trace anomaly of the stress tensor for the scalar fields moving in the background of a (3+1) dimensional curved manifold is given by [39, 40]

$$\langle T^\mu{}_\mu\rangle^{(1)} = \frac{1}{2880\pi^2}\left(R_{abcd}R^{abcd} - R_{ab}R^{ab} + \nabla_a\nabla^a R\right)\ . \qquad (18)$$

For the metric (2), the invariant scalars are explicitly found to be

$$\begin{aligned}
R_{abcd}R^{abcd} &= \frac{8}{r^8}(7Q^4 - 12MrQ^2 + 6M^2r^2) \\
&\quad -\frac{e^{-r^2/(4\theta)}}{4\pi r^6\theta^3}(\sqrt{2}Q^2 - 4M\sqrt{\pi\theta})\left[(3Q^2-4Mr)r^4 + 4\theta r^2(Q^2-4Mr)\right] \\
R_{ab}R^{ab} &= \frac{4Q^4}{r^8} + \frac{MQ^2e^{-r^2/(4\theta)}}{\sqrt{\pi}\theta^{5/2}r^6}(r^4 - 2\theta r^2) \\
R &= \frac{e^{-r^2/(4\theta)}(Q^4 - 4MQ^2\sqrt{2\pi\theta} + 8M^2\pi\theta)(r^4 - 10r^2\theta + 8\theta^2)}{4r^2\theta^3(\sqrt{2}\pi Q^2 - 4M\pi^{3/2}\sqrt{\theta})}\ . \qquad (19)
\end{aligned}$$

Note that in the commutative limit ($\theta \to 0$), the above results match with the known results of the standard vacuum RN spacetime metric [42], for which $R_{abcd}R^{abcd} = \frac{8}{r^8}(7Q^4 - 12MrQ^2 + 6M^2r^2)$, $R_{ab}R^{ab} = \frac{4Q^4}{r^8}$, $R = 0$. To find the trace anomaly (18), we now evaluate

$$\begin{aligned}
\nabla_a\nabla^a R &= \frac{e^{-r^2/(4\theta)}}{32\pi r^6\theta^5}(\sqrt{2}Q^2 - 4M\sqrt{\pi\theta})\left[r^8(Q^2 - 2Mr + r^2) - \alpha_1 + \alpha_2\right] \qquad (20) \\
&\text{where,}\quad \alpha_1 = 4r^6\theta[5Q^2 + r(6r - 11M)] \\
&\qquad\qquad \alpha_2 = 4r^4\theta^2(9Q^2 - 32Mr + 23r^2)\ .
\end{aligned}$$

Exploiting all these results, the trace anomaly is computed from (18), upto the leading order in $\mathcal{O}(e^{-r^2/(4\theta)})$, as

$$\begin{aligned}\langle T^{\mu}{}_{\mu}\rangle^{(1)} &= \frac{1}{2880\pi^2}\left[\frac{52Q^4}{r^8}-\frac{96MQ^2}{r^7}+\frac{48M^2}{r^6}\right.\\ &\left.+\frac{e^{-r^2/(4\theta)}}{32\pi r^4\theta^5}(\sqrt{2}Q^2-4M\sqrt{\pi\theta})\alpha_3-\frac{MQ^2e^{-r^2/(4\theta)}}{\sqrt{\pi}\theta^{5/2}r^6}(r^4-2\theta r^2)\right] \qquad (21)\end{aligned}$$

where,

$$\begin{aligned}\alpha_3 &= r^6(Q^2-2Mr+r^2)-4r^4\theta(5Q^2+6r^2-11Mr)\\ &+4\theta^2r^2(3Q^2-24Mr+23r^2)-32\theta^3(Q^2-4Mr)\ . \qquad (22)\end{aligned}$$

Substituting this in (16) and performing the integral yields

$$\begin{aligned}\tilde{\beta}_1 &= \frac{(r_0-M)}{2880\pi^2\omega}\beta\left[\frac{52Q^4}{5r_h^5}-\frac{24MQ^2}{r_h^4}+\frac{16M^2}{r_h^3}+\frac{2MQ^2e^{-r_0^2/(4\theta)}}{\theta\sqrt{\pi\theta}r_0^3}(2\theta-r_0^2)\right.\\ &\left.+\frac{e^{-r_0^2/(4\theta)}}{32\pi}(\sqrt{2}Q^2-4M\sqrt{\pi\theta})\alpha_4\right] \qquad (23)\end{aligned}$$

where ,

$$\alpha_4=\frac{2Q^2}{\theta^2}\left(\frac{r_0^3}{\theta^2}-\frac{16}{r_0}-\frac{14r_0}{\theta}-\frac{32\theta}{r_0^3}\right)+\left(\frac{2r_0^4}{\theta^4}-\frac{28r_0^2}{\theta^3}\right)(r_0-2M)+\frac{16}{\theta^2}(r_0+2M)+\frac{256M}{\theta r_0^2}\ . \qquad (24)$$

To compute ω, we calculate the Komar energy integral (17). For the spacetime metric (2) one has the following expressions for the Killing vectors (K^μ), its inverse (K_ν) and the unit normal vectors (n^μ, σ^ν)

$$K^\mu=(1,0,0,0),\quad K_\nu=-f_\theta(1,0,0,0) \qquad (25)$$

$$n^\mu=f_\theta^{-1/2}(1,0,0,0) \qquad (26)$$

$$\sigma^\nu=f_\theta^{1/2}(0,1,0,0) \qquad (27)$$

$$\sqrt{p^{(2)}}=r^2\sin\theta. \qquad (28)$$

Using these, eq.(17) is simplified as

$$\omega = \frac{1}{8\pi}\int_{\tilde{\theta}=0}^{\pi}\int_{\phi=0}^{2\pi} r^2 \sin\tilde{\theta}\ (\partial_r f_\theta)\ d\tilde{\theta}d\phi\ . \tag{29}$$

Finally, integrating over the angular variables, near the event horizon (4), the above expression for ω simplifies to

$$\begin{aligned}\omega &= (r_0 - M)\left[1 - \frac{Mr_0}{\sqrt{\pi\theta}(r_0-M)}\left(1+\frac{r_0^2}{2\theta}\right)e^{-r_0^2/(4\theta)}\right.\\ &\left. + \frac{Q^2 r_0^3}{\sqrt{24\pi\theta^2}(r_0-M)}e^{-r_0^2/(4\theta)} + \frac{Q^2}{\sqrt{\pi\theta}(r_0-M)^2}\left(2M - \frac{Q^2}{\sqrt{2\pi\theta}}\right)e^{-r_0^2/(4\theta)}\right]\ . \end{aligned} \tag{30}$$

Substituting this in the expression for $\tilde{\beta}_1$ in eq.(23), we obtain :

$$\tilde{\beta}_1 = \frac{r_0^2}{1440\pi(r_0-M)}\left[\alpha_5 + \left(\frac{52Q^4}{5r_0^5} - \frac{24MQ^2}{r_0^4} + \frac{16M^2}{r_0^3}\right)\alpha_6\right] \tag{31}$$

where,

$$\begin{aligned}\alpha_5 &= \frac{52Q^4}{5r_h^5} - \frac{24MQ^2}{r_h^4} + \frac{16M^2}{r_h^3} + \frac{2MQ^2}{\theta\sqrt{\pi\theta}r_0^3}(2\theta - r_0^2)e^{-r_0^2/(4\theta)}\\ &+\frac{1}{32\pi}(\sqrt{2}Q^2 - 4M\sqrt{\pi\theta})e^{-r_0^2/(4\theta)}\left[\frac{2Q^2}{\theta^2}\left(\frac{r_0^3}{\theta^2} - \frac{16}{r_0} - \frac{14r_0}{\theta} - \frac{32\theta}{r_0^3}\right)\right.\\ &\left.+\left(\frac{2r_0^4}{\theta^4} - \frac{28r_0^2}{\theta^3}\right)(r_0 - 2M) + \frac{16}{\theta^2}(r_0+2M) + \frac{256M}{\theta r_0^2}\right]\\ \alpha_6 &= \frac{Mr_0}{\sqrt{\pi\theta}(r_0-M)}\left(1+\frac{r_0^2}{2\theta}\right)e^{-r_0^2/(4\theta)} - \frac{Q^2r_0^3}{\sqrt{24\pi\theta^2}(r_0-M)}e^{-r_0^2/(4\theta)}\\ &-\frac{Q^2}{\sqrt{\pi\theta}(r_0-M)^2}\left(2M - \frac{Q^2}{\sqrt{2\pi\theta}}\right)e^{-r_0^2/(4\theta)}\ . \end{aligned} \tag{32}$$

Finally using eq(s)(15) and (31), we find the cherished result for the corrected entropy/area law (upto leading order in θ) for the NC inspired RN black hole

$$\begin{aligned} S_{bh} &= \frac{A_\theta}{4\hbar} + \frac{r_0^2}{720(r_0-M)}\left[\alpha_5 + \left(\frac{52Q^4}{5r_0^5} - \frac{24MQ^2}{r_0^4} + \frac{16M^2}{r_0^3}\right)\alpha_6\right]\ln\frac{A_\theta}{\hbar} + \mathcal{O}(\sqrt{\theta}e^{-\frac{M^2}{\theta}})\\ &= S_{BH} + \frac{r_0^2}{720(r_0-M)}\left[\alpha_5 + \left(\frac{52Q^4}{5r_0^5} - \frac{24MQ^2}{r_0^4} + \frac{16M^2}{r_0^3}\right)\alpha_6\right]\ln S_{BH} + \mathcal{O}(\sqrt{\theta}e^{-\frac{M^2}{\theta}}). \end{aligned} \tag{33}$$

This is the general expression for the entropy of NC inspired RN black hole where both the

NC and quantum effects have been taken into account. The first term in this expression is the semiclassical entropy and the next term is the leading correction. It is logarithmic in nature. The coefficient of the logarithmic correction is different from the standard RN black hole [35, 42] due to the presence of noncommutative parameter (θ). In the commutative limit $\theta \to 0$, the expression for the corrected entropy exactly matches with the standard RN case where the coefficient of the leading correction is $\frac{1}{90}$, obtained in the path integral [39], euclidean [43] and tunneling [35],[42] formalisms. Also, in the $Q \to 0$ limit, eq.(33) reduces to the entropy of the NC inspired Schwarzschild black hole [13].

We conclude by making the following comments. In this paper we have once again emphasized the importance of the Voros star product in writing down the mass and charge densities of a NC inspired RN black hole. To point out the role played by the Voros product, we need to take recourse to a rigorous formulation of NC quantum mechanics [17],[18] as in our earlier paper [13].

We have then studied the status of the entropy/area law along with corrections of the NC inspired RN black hole. A general result for this black hole entropy/area law was found, taking both quantum and NC effects into account. For this we first used the tunneling method by going beyond the semiclassical approximation and calculated the corrected Hawking temperature. Using this modified temperature and the first law of black hole thermodynamics we then calculated the corrected entropy. The (NC) semiclassical Bekenstein-Hawking value was reproduced at the next to leading order in θ and higher order correction contained logarithm of horizon area. The coefficient of the logarithmic term was fixed by using the trace anomaly of the scalar field stress tensor. The trace anomaly and the Komar energy integral for the NC inspired RN metric were explicitly calculated to determine this coefficient. The value of the coefficient was found to have NC correction. We also show that the commutative limit of the corrected entropy/area law of this black hole matches with the standard result for the RN black hole [39, 43, 35].

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